

## 1 Example 6 — A Convex Sparse Example

$$\min_{x \in \mathbb{R}^n} \quad \frac{a}{4} \sum_{i=1}^n (x_i - 1)^4 \quad (1)$$

$$\text{s.t.} \quad 4ax_1 + 2ax_2 = 10a \quad (2)$$

$$5a \leq 2ax_1 + ax_3 \quad (3)$$

$$a \leq 2ax_1 + 0.5ax_i \leq 2an \quad (4)$$

$$x_1 \text{ free,} \quad (5)$$

$$0.0 \leq x_2, \quad (6)$$

$$1.5 \leq x_3 \leq 10 \quad (7)$$

$$0.5 \leq x_i, \quad \forall i = 4, \dots, n \quad (8)$$

Here  $n \geq 3$  and  $a > 0$  are parameters that can be modified via Ex6 driver's command arguments. Their default values are  $n = 3$  and  $a = 1.0$ .

The analytical optimal solution is  $x_1 = 1.75$ ,  $x_2 = x_3 = 1.5$ , and  $x_i = 1$  for  $i \in \{4, 5, \dots, n\}$ . The objective value is 0.110352. The file `nlpSparse_ex6_driver.cpp` provides more details about how to use HiOp to solve instances of this example/test problem.

## 2 Example 7 — A Nonconvex Sparse Example

$$\min_{x \in \mathbb{R}^n} \quad -\frac{a}{4} \sum_{i=1}^n (x_i - 1)^4 + 0.5 \sum_{i=1}^n x_i^2 \quad (9)$$

$$\text{s.t.} \quad 4x_1 + 2x_2 = 10 \quad (10)$$

$$4x_1 + 2x_2 = 10 \quad (11)$$

$$5 \leq 2x_1 + x_3 \quad (12)$$

$$4x_1 + 2x_3 \leq 19 \quad (13)$$

$$1 \leq 2x_1 + 0.5x_i \leq 2n \quad (14)$$

$$x_1 \text{ free,} \quad (15)$$

$$0.0 \leq x_2, \quad (16)$$

$$1.5 \leq x_3 \leq 10 \quad (17)$$

$$0.5 \leq x_i, \quad \forall i = 4, \dots, n \quad (18)$$

Here  $n \geq 3$  and  $a > 0$  are parameters which can be tuned via Ex6 driver's command arguments. Their default values are  $n = 3$  and  $a = 0.1$ .

Note that the equality constraints (10) and (11) are duplicate. This is done on purpose to make the constraint Jacobian matrix rank deficient and to stress test HiOp on this numerically difficult, nonconvex problem. The file `nlpSparse_ex7_driver.cpp` provides more details about how to use HiOp in various configurations to solve instances of this example/test problem.